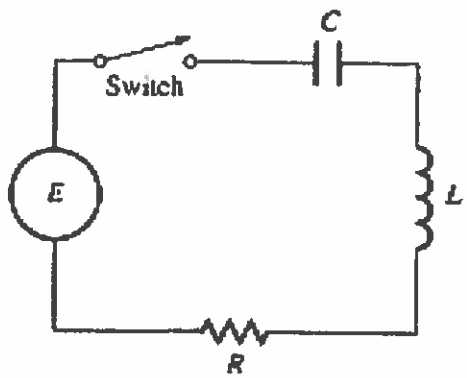
3.7: Electrical Circuits



Circuit Element	Voltage Drop
Inductor	$L\frac{dI}{dt}$
Resistor	RI
Capacitor	$\frac{1}{C}Q$

Here we examine an RLC electric circuit which consists of

1. A **resistor** with a resistance of R ohms.
2. An **inductor** with an inductance of L henries, and
3. A **capacitor** with a capacitance of C farads.

These objects are in a series after the electromotive force (such as a battery of generator) that supplies a voltage of $E(t)$ volts at time t. If the switch is closed, this results in a current of $I(t)$ amperes in the circuit and a charge of $Q(t)$ coulombs on the capacitor at time t. The relation between Q and I is given by

$$\frac{dQ}{dt} = I(t). \tag{1}$$

According to elementary properties of electricity, the **voltage drops** across the three circuit elements are shown above. With the help of Kirchoff's Law we arrive at the differential equation

$$L\frac{dI}{dt} + RI + \frac{1}{C}Q = E(t). \tag{2}$$

Substituting (2) into (1) we get the equation

$$LQ'' + RQ' + \frac{1}{C}Q = E(t) \tag{3}$$

for the charge Q, or more commonly,

$$LI'' + RI' + \frac{1}{C}I = E'(t) \tag{4}$$

for the current I.

Question 1. Does this look familiar to the mass-spring-dashpot mechanical system

$$mx'' + cx' + kx = F(t)$$

from Section 3.4 and 3.6? What transformations would you make to create a **mechanical-electrical analogy**?

Mass (m) = Inductance (L)

Damping Constant (c) = Resitance (R)

Spring Constant (k) = Reciprical capacitance $(1/C)$

Position (x) = Charge (Q) / Current (I)

Force (F) = Electromotive Foree (E) or (E').

Question 2. Why would this analogy be useful in practice?

When it is expensive, impractical or dangerous to measure ~~#~~ displacement or velocity.

Analog Computers tested models for first nuclear reactors.

Example 1. Consider an RLC circuit with $R = 50$ ohms (Ω), $L = 0.1$ henry (H), and $C = 5 \times 10^{-4}$ farad (F). At time $t = 0$, when both $I(0)$ and $E(0)$ are zero, the circuit is connected to a 110-V, 60-Hz alternating current generator. Find the current in the circuit and the time lag of the steady periodic current behind the voltage.

60 Hz means $\omega = (2\pi)(60)$ rad/s $\simeq 377$ rad/s

So $E(t) = 110 \sin 377t$. Then

$$(0.1)I'' + 50I' + 2000I = 377 \cdot 110 \cos 377t.$$

Thus

$$I_p = \frac{110}{59.58} \cos(377t - 0.575)$$

$$I_c = (-0.307)e^{-44t} + (1.311)e^{-456t}$$

Homework. 1, 7, 11-15 (odd)